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Stochastic Consumer Behavior and Brownian Motion.

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ABSTRACT

Previous researches suggested multiple approaches in modeling consumer purchase behavior. Along with uncertainties, in many situations, particularly in low involvement products and frequently purchased consumer packaged goods, little conscious decision making takes place. We assume that stochastic purchase behavior observed in the consumers is due to some kind of agitation within the consumers before or during the purchase. Due to such agitation in the mind of the consumers, purchase decision of above products experience several internal forces in different directions. We show that these forces imbalance consumers' mind and purchase decision become random. Since these forces are haphazard, we show that resultant force that influences the purchase decision is also haphazard. The main purpose of this article is to show that one can represent consumers' purchase behavior of low involvement and frequently purchased products as Brownian motion. In other words, we are to prove the existence of the process by conforming above four defined properties of Brownian motion.

Previous researches suggested multiple approaches in modeling consumer purchase behavior. Along with uncertainties, in many situations, particularly in low involvement products and frequently purchased consumer packaged goods, little conscious decision making takes place. In such situations stochastic model – concentrating on random nature of choice becomes more appropriate than deterministic approach. Another reason that stochastic choice model is suitable for such goods is availability of large volume of brand switching data with market researchers (Lilien, Kotler and Murthy 1992, Geertz 1978).

We assume that stochastic purchase behavior observed in the consumers is due to some kind of agitation within the consumers before or during the purchase. Due to such agitation in the mind of the consumers, purchase decision of above products experience several internal forces in different directions. Consequently, these forces imbalance consumers' mind and purchase decision become random. Since these forces are haphazard, resultant force that influences the purchase decision is also haphazard. The smaller the involvement, the larger is the resultant force and consequently more irregular the movements are (He 2008, Liang Pokharel, S., & Lim, G. H. (2009).).

The basic concept on which kinetic theory stands is the motion of molecules due to thermal agitation. Although kinetic theory explains many thermal and allied phenomena, direct experimental evidence of random motion of molecules came only after the theory of Robert Brown and termed as Brownian motion. In 1900, Bachelier exhibited Markovian nature of Brownian motion (Kao 1997). According to such phenomenon position of a particle at time (t+s) depends on its position at time t and does not depend on its position before time t. Markov model in stochastic consumer choice behavior assumes impact of just previous purchase on present purchase. This can be extended to post purchase behavior of the consumer for next purchase. Hence this assumption is similar to Bachelier's exhibition of Markovian nature of Brownian motion (Bertola & Drazen 1991).

This paper introduces and implements mathematical approach of Brownian motion in consumers' purchase decision (D_t) in low involvement product. It is assumed that this kind of stochastic process is a continuous time stochastic process $\{D_t: 0 \leq t < T\}$. It is called a standard Brownian motion on (0,T) if it has the following properties.

1) $D_0 = 0$
2) The increments of D_t are independent: that is for any finite sets of times.

   $0 \leq t_1 < t_2 < t_3 \ldots\ldots < t_n < T$ then the random variables

   $D_{t2} – D_{t1}, D_{t3} – D_{t2}, D_{t4} - D_{t3}, \ldots\ldots D_{tn} – D_{t(n-1)}$ are independent.
3) For any $0 \leq c < t < T$, the increment $D_t – D_c$ has a Gaussian distribution with mean zero and variance (t-c).

4) For all α in a set of probability one, D_t (α) is a continuous function of ‘t’.

The main purpose of this article is to show that one can represent consumers’ purchase behavior of low involvement and frequently purchased products as Brownian motion. In other words, we are to prove the existence of the process by conforming above four defined properties of Brownian motion.

With a view to provide support to our hypothesis that construction of Brownian motion is true, we are required to use multivariate Gaussian distribution (Lin & Sibdari 2009). The most critical factor of multivariate Gaussian is that their joint density is fully determined by mean vector and variance-covariance matrix (Raman & Naik 2004, teele, 2000). Hence we are required to identify tools that help us check that agitations of our processes are Gaussian distribution as well as the agitations are independent. As per the properties of multivariate Gaussian, if B is a d-dimensional random vector, then

Mean vector of **B** = $\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \cdots \\ B_d \end{bmatrix}$ is a vector $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \cdots \\ \mu_d \end{bmatrix}$

And the varience-covarience matrix of B is given by

$$\Sigma = \begin{bmatrix} \sigma_{11}\sigma_{12}\sigma_{13}........\sigma_{1d} \\ \sigma_{21}\sigma_{22}\sigma_{23}........\sigma_{2d} \\ \sigma_{31}\sigma_{32}\sigma_{33}........\sigma_{3d} \\ .... \\ .... \\ \sigma_{d1}\sigma_{d2}\sigma_{d3}..........\sigma_{dd} \end{bmatrix}$$

$\sigma_{ij} = E\,[(B_i - \mu_i)\,(B_i - \mu_i)]$ and

$$f\,(B) = \frac{1}{\sqrt[d]{2\pi}}[\exp\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\,\}] \qquad \text{for all} \quad x \in \mathbf{R}^d$$

The profile of Brownian motion in this case can be represented as at each instant in time, the consumer randomly chooses a brand and then purchases that brand. This approach is both intuitive and rigorous. We assume each time of purchase is a combination of several decisions taken at multiple of Δt. In each of such instant, the consumer randomly chooses to buy or not to buy one brand and goes to second. We can express that if a consumer starts from a particular point, in every Δt time it covers a distance of Δx from starting point. To model this randomness, we consider a sequence of identically distributed random variables (Y_i, i >1) such that $P(Y_i = \Delta x) = P(Y_i = -\Delta x) = ½$. (Lin & Sibdari 2009)

At time t, the purchaser to make [t / Δt] moves (where [g] denotes the integer part of g). The respondents position will be $U_t = Y_1+Y_2+Y_3+Y_4+\ldots Y_{[t/\Delta t]}$. All this takes place on a very small scale at the time of purchase or just before purchase. We would like to assume both Δt and Δx tend to zero in an appropriate way. Note that $EU^2 \approx (\Delta x)^2 . (t / \Delta t)$. In order for this expression to have a limit, we must consider that $(\Delta x)^2 / \Delta t$ have a limit. The increment Δt will be very small and Δx will also be small so that $(\Delta x)^2$ will also be very small. The most appropriate choice is $\Delta x = (\Delta t)^{1/2}$ and Δt = 1/n, where 'n' is integer.

The true formulation of this approach can be represented as follows:

On a probability space (Ω, Ψ, P), let

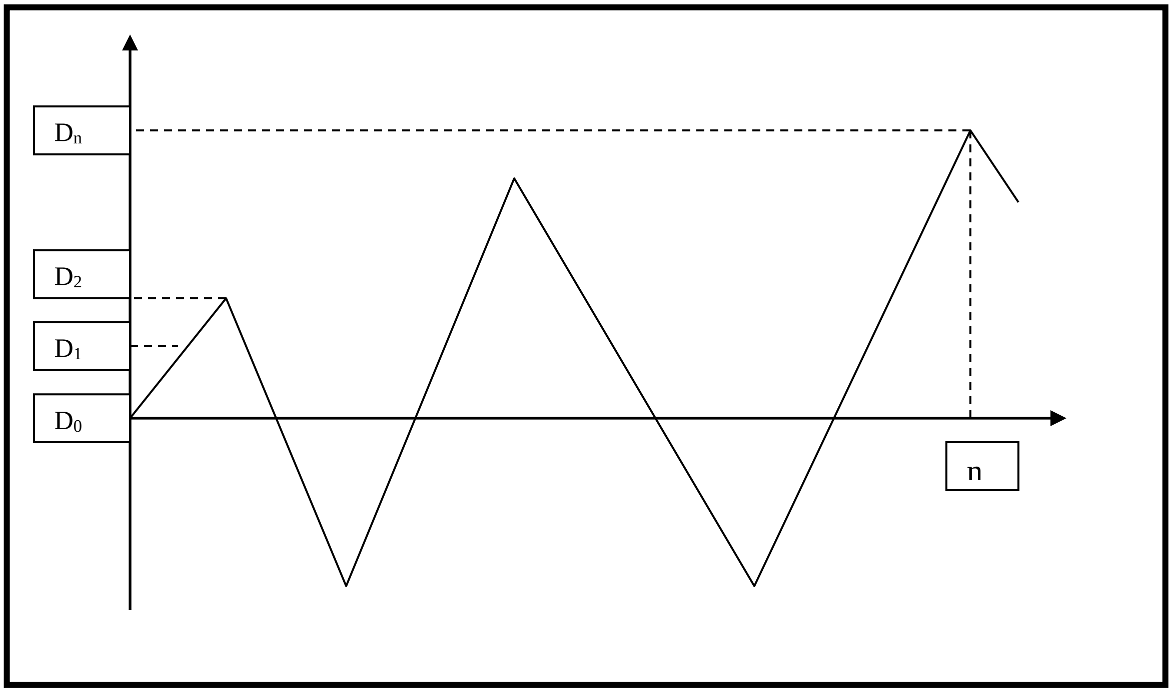
$$P(X_i = 1) = P(X_i = -1) = ½ , \qquad \text{for } i \in \mathbf{N}$$

Be a group of identically distributed random variables (the X_i are said to be independent random variables). To this group, we express the sequence (D_n, n ≥ 0) defined by

$$D_0 = 0$$

$$D_n = \Sigma_{i=1}^{n} X_i$$

We have $E(D_n) = 0$ and Var $(D_n) = n$. we can say that the sequence is a *random walk*. We can illustrate it as a game of tossing a coin. The player gains $1 if it comes up head and looses $1 if tails appear. Let us start with that the player has no initial wealth ($D_0 = 0$). His capital at time n (after n tosses) is D_n. If we draw the results of N successive tosses, we can plot the outcomes as below:



Random walk

It is to be noted that the sequence (D_m – D_n) at m ≥ n is independent of (D_0 , D_1, D_2, ….. D_n). Here D_m – D_n has the same probability law as D_{m-n} as the binomial distribution depends only on (m-n).

Let us follow a two stage normalization. Let N be fixed. In the first stage we transform the time interval [0,N] into an interval [0,1] and in the second stage, we change the scale of values taken by D_n. hence, actually we define a group of random variables bt real numbers of the form k/N for k∈ **N.**

$$U_{k/N} = \frac{1}{\sqrt{N}} D_k$$

We move from $U_{\frac{k}{N}}$ to $U_{\frac{k+1}{N}}$ in a very small time interval 1/N by making a small displacement $\frac{1}{\sqrt{N}}$ in any of the two directions). We have

$$E\,(U_{\frac{k}{N}}) = 0 \quad \text{and } Var\,(U_{\frac{k}{N}}) = k/N.$$

The independency and stationary properties of random walk still holds Martínez-de-Albéniz & Talluri 2011).

Hence U_N converges to a process B that has continuous path (i.e. for almost all ω, the mapping $t \rightarrow B_t(\omega)$ is continuous) and which satisfies

1. $B_0 = 0$
2. $B_{t+s} - B_t$ has normal distribution with N (0, s)
3. $B_{t+s} - B_t$ is independent of $B_{t(i)} - B_{t(i+1)}$ for $t_0 < t_1 < t_2 <\ldots\ldots..< t_n = t$

Brownian motion is the only process that satisfies (1) and (3) above. To show that the distribution depends only on s, we introduce the notation $\Delta B(t) = B(t+\Delta t) - B(t)$ where $B(t) = B_t$ and $\Delta t > 0$. The Brownian motion then satisfies

$$E\,[\Delta B\,(t)] = 0 \qquad Var\,[\Delta B\,(t)] = \Delta t \quad \ldots\ldots \text{ using (2)}$$

$$E_t\,[\Delta B\,(t)] = 0 \qquad E_t\,[(\Delta B\,(t))^2] = \Delta t \quad \ldots\ldots\text{using (2) and (3)}$$

Where E_t is the conditional expectation with respect to $\Psi_t = \sigma (B_s , s \leq t)$. The equality $E_t (\Delta B (t)) = 0$ can be interpreted as if the position of the Brownian motion at time t is known, then averahe move between time t and t+ Δt is zero. This property is the result of independency and Gaussian nature of Brownian motion.

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